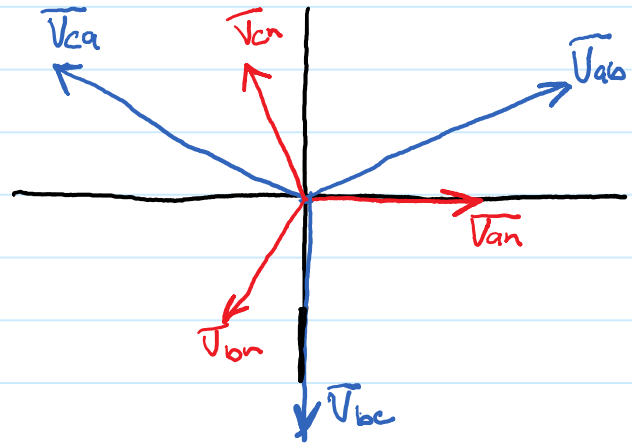
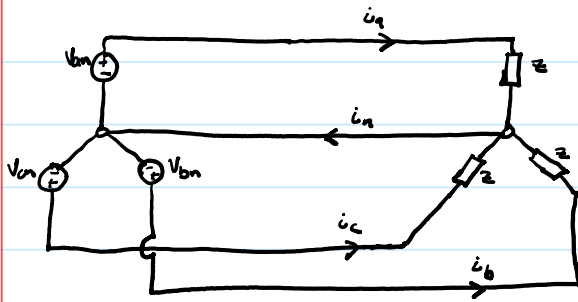


2018-09-07-1

$$\begin{aligned}\bar{V}_{an} &= V_{\phi} \angle 0^\circ \\ \bar{V}_{bn} &= V_{\phi} \angle -120^\circ \\ \bar{V}_{cn} &= V_{\phi} \angle 120^\circ\end{aligned}$$



What is voltage from a-b?

$$\begin{aligned}\bar{V}_{ab} &= \bar{V}_{an} - \bar{V}_{bn} \Rightarrow \bar{V}_{ab} = \bar{V}_{an} + (-\bar{V}_{bn}) \\ &= V_{\phi} \angle 0^\circ + (V_{\phi} \angle (-120^\circ + 180^\circ)) \\ &= V_{\phi} + V_{\phi} \cos(60^\circ) + jV_{\phi} \sin(60^\circ) \\ &= V_{\phi} \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right)\end{aligned}$$

$$|\bar{V}_{ab}| = V_{\phi} \sqrt{\frac{9}{4} + \frac{3}{4}} \Rightarrow |\bar{V}_{ab}| = V_{\phi} \sqrt{3}$$

$$\theta_{ab} = \tan^{-1}\left(\frac{\sqrt{3}/2}{3/2}\right) \Rightarrow \theta_{ab} = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \theta_{ab} = 30^\circ$$

$$\boxed{\bar{V}_{ab} = V_{\phi} \sqrt{3} \angle 30^\circ}$$

Can verify:

$$\begin{aligned}\bar{V}_{bc} &= V_{\phi} \sqrt{3} \angle -90^\circ \\ \bar{V}_{ca} &= V_{\phi} \sqrt{3} \angle 150^\circ\end{aligned}$$

Line to line Voltage (aka line voltage): $V_L = V_{\phi} \sqrt{3}$

What are the currents in each phase/line?

$$\bar{Z} = Z \angle \theta$$

$$\bar{I}_a = \frac{V_{an} \angle 0^\circ}{Z \angle \theta} \Rightarrow \bar{I}_a = \frac{V_{\phi}}{Z} \angle -\theta \Rightarrow \bar{I}_a = I_L \angle -\theta \Rightarrow \boxed{\bar{I}_a = I_L \angle -\theta}$$

Can verify:

$$\begin{aligned}\bar{I}_b &= I_L \angle -120^\circ - \theta \\ \bar{I}_c &= I_L \angle 120^\circ - \theta\end{aligned}$$

$$\begin{aligned}\bar{I}_n &= \bar{I}_a + \bar{I}_b + \bar{I}_c \Rightarrow \bar{I}_n = I_L \left(\cos(-\theta) + j\sin(\theta) + \cos(-\theta - 120^\circ) + j\sin(-\theta - 120^\circ) + \cos(-\theta + 120^\circ) + j\sin(-\theta + 120^\circ) \right) \\ &= I_L \left(\cos(\theta) + 2\cos(120^\circ)\cos(\theta) + j(\sin(\theta) - 2\cos(120^\circ)\sin(\theta)) \right) \\ &= I_L \left(\cos(\theta) - \cos(\theta) + j(-\sin(\theta) + \sin(\theta)) \right)\end{aligned}$$

$$\boxed{\bar{I}_n = 0}$$

* For balanced loads, $\bar{I}_n = 0$. So, can get rid of wire.

2018-09-07-2

Complex power:

$$\bar{S} = \bar{S}_a + \bar{S}_b + \bar{S}_c + \bar{S}_n$$

$$\bar{S}_n = 0 \text{ because } \bar{I}_n = 0$$

$$\bar{S} = \bar{S}_a + \bar{S}_b + \bar{S}_c$$

$$\bar{S} = \bar{V}_{an} \bar{I}_a^* + \bar{V}_{bn} \bar{I}_b^* + \bar{V}_{cn} \bar{I}_c^*$$

$$= V_\phi I_L \angle \theta + V_\phi I_L \angle (120^\circ + \theta + 120^\circ) + V_\phi I_L \angle (120^\circ + \theta - 120^\circ)$$

$$\boxed{\bar{S} = 3V_\phi I_L \angle \theta}$$

* 3x power as 1 ϕ

* $\frac{3}{2}$ x wires of 1 ϕ (3 vs. 2) Better than 2 ϕ (3 vs. 4)

$$V_L = V_\phi \sqrt{3} \Rightarrow V_\phi = \frac{V_L}{\sqrt{3}}$$

$$\boxed{\begin{aligned} \bar{S} &= \sqrt{3} V_L I_L \angle \theta \\ P &= \sqrt{3} V_L I_L \cos(\theta) \\ Q &= \sqrt{3} V_L I_L \sin(\theta) \end{aligned}}$$

Ex | Wye connected load

$$P = 24 \text{ kW} \quad \text{PF} = 0.8 \text{ lag} \quad V_L = 480 \text{ V}$$

Find: a) I_ϕ and V_ϕ

b) \bar{S}_{total}

$$a) V_\phi = \frac{V_L}{\sqrt{3}} \Rightarrow \boxed{V_\phi = 277.13 \text{ V}}$$

$$P = \sqrt{3} V_L I_L (\text{PF})$$

$$I_L = \frac{P}{\sqrt{3} V_L (\text{PF})} \Rightarrow \boxed{I_L = 36.08 \text{ A}}$$

$$b) \theta = \cos^{-1}(\text{PF}) \Rightarrow \theta = 36.87^\circ$$

$$\bar{S} = \sqrt{3} V_L I_L \angle \theta$$

$$\boxed{\begin{aligned} \bar{S}_{\text{tot}} &= 30,000 \angle 36.87^\circ \text{ VA} \\ &= 24,000 + j18,000 \text{ VA} \end{aligned}}$$